

This exam consists of 2 questions. Please answer both.

1. Are the following statements true, false or neither, and why? Explain your answer in detail. You do not need to use math for these questions but should feel free to if it helps you be precise.
 - (a) Empirically, countries as a whole tend to save in recessions and borrow in booms. This makes sense from the point of view of a theory of intertemporal consumption smoothing.

Solution:

There is more than one way to answer this question and receive full points. The following is one suggestion.

The empirical statement is true, the theoretical statement is false. Empirically, countries run current account and trade balance surpluses in recessions, but accumulate net foreign debt and borrow in booms. In terms of theory, a standard small open endowment economy model predicts that in relatively good times, households prefer to save abroad, and in bad times borrow against future income. The mechanism is standard intertemporal consumption smoothing in the face of persistent but stationary income shocks: It is optimal to adjust consumption less than 1 for 1 to changes in expected lifetime income, the remainder is saved. Since the only savings vehicle is external debt in the endowment economy, the trade balance improves.

We can relax the assumptions of this framework/ consider extensions that can make sense of a countercyclical trade balance, including but not limited to:

- An investment channel. This doesn't change the basic counterfactual logic of the consumption smoothing channel, but since the current account is given by the difference between savings and investment, sufficiently procyclical investment can outweigh the consumption smoothing channel and lead to an overall countercyclical current account.

In a small open economy this will be the case if adjusting investment is not too costly. We discussed a two country model where domestic investment demand rises sufficiently in response to positive domestic productivity shocks to generate a countercyclical trade balance because of home bias in consumption and specialization in tradables.

- Very persistent/ permanent shocks reduce the consumption smoothing incentive that generates a procyclical trade balance. If shocks are permanent, agents adjust consumption one-for-one since there is no room to achieve a smoother consumption profile over time.
- Frictions like sovereign default risk coupled with consumption smoothing motives can rationalize countercyclical current accounts and interest rates. Incentives to default are higher in recessions (and with high levels of debt), so investors demand a higher return to compensate for

this risk, and the price at which a sovereign can sell his bonds falls. If it is sufficiently expensive to borrow in downturns, the theory can account for why countries save in recessions rather than borrow.

- (b) International equity portfolios are not diversified enough.

Solution:

There is more than one way to answer this question and receive full points. The following is one suggestion.

This question asks about the empirical make up of country's stock portfolios, and what theories we have to explain them. In the data, portfolios have a strong home bias. This is puzzling in the simplest theory, but can be rationalized if we consider the effects of investment and intermediate goods.

In a world with two perfectly symmetric countries where labor is not a factor of production and that are subject to idiosyncratic shocks, it is optimal to hold a constant portfolio with each country holding a claim on 50 percent of the other country's output, in other words a constant portfolio with 50 percent of domestic and 50 percent of foreign shares (each country's share in the world portfolio is the same since they are perfectly symmetric). This is optimal because it hedges country-specific risk. In good times at home, domestic firms make dividend payments to foreigners, but in recessions, domestic residents in turn receive payments from abroad.

If labor is a factor of production, there is an optimal constant portfolio that involves *shorting* domestic stock. The reason is the following. Domestic labor income is non-diversifiable so households will use dividend income to diversify any labor income risk. With a standard Cobb Douglas production function, domestic dividend income and labor income are highly correlated, so it becomes optimal to short domestic stock and hold more than 100 percent in foreign shares. This is in very strong contrast to the data where we observe home rather than foreign bias in equities.

We can derive this more formally as follows. Consider two perfectly symmetric countries that produce the single tradable consumption good using a Cobb Douglas production technology with capital and labor, both of which are immobile across countries. Households have log preferences over consumption and leisure. Under the log utility assumption, complete risk sharing implies that the difference between country consumption levels is zero: $\Delta c \equiv c - c^{(*)} = 0$.

If θ is the capital share in production, households receive labor income $1 - \theta y$ and domestic and foreign capital income, θy and $\theta y^{(*)}$, respectively. Let λ denote the share of domestic stocks held by domestic households, and by symmetry and market clearing $1 - \lambda$ the foreign portfolio share. Without investment, we can derive the constant optimal foreign portfolio share as

$$1 - \lambda = \frac{1}{2\theta}$$

If $\theta = 1$ (Lucas), the optimal share is 0.5, the higher the capital share, the higher the *foreign* bias (Baxter Jermann).

We can rationalize home bias close to that observed in the data in a theory that includes investment in physical capital and differentiated goods. Consider a two country two good model with perfect specialization in intermediate goods production. The final consumption good is a Cobb Douglas aggregate of country specific intermediates. Intermediates are produced with domestic capital. It is optimal to be home biased in this setting for the following reason. A positive productivity shock at home will increase relative investment in the domestic capital stock because it is more efficient to produce home intermediates. Relative domestic dividends fall. Relative domestic labor income on the other hand rises: The home shock tends to appreciate the terms of trade and make domestic labor income more valuable. The terms of trade appreciate because of the increase in demand for the domestic final good to use for investment. If home bias in production of the final good is large this implies a strong increase in demand for the domestic intermediate. This effect needs to dominate the tendency for the ToT to depreciate due to the abundant supply of domestic intermediates after the shock. If that is the case, then domestic dividends and labor income are negatively correlated, and thus domestic stocks provide a good hedge against labor income fluctuations. It is optimal to be home bias in equities.

2. Consider a global endowment economy that consists of two perfectly symmetric countries. Denote foreign country variables with stars. There are two consumption goods, and there is perfect specialization in their supplies: The home country is endowed with the world supply of X and the foreign country is endowed with the world supply of Y . Both countries consume a mix of the two goods, specifically home country residents maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t u(x_t, y_t)$$

where

$$u(x, y) = \frac{(x^\theta y^{1-\theta})^{1-\sigma}}{1-\sigma}$$

and foreign country residents maximize the same function of their own consumption, $u(x^*, y^*)$. There are no international transport or trade costs.

- (a) Set up the social planners problem and solve it. Show that the allocations are given by

$$\begin{aligned} x &= \omega X \\ y &= \omega Y \\ x^* &= (1 - \omega) X \\ y^* &= (1 - \omega) Y \end{aligned}$$

where μ is the social planner's weight on the home country and $\omega(\mu)$ is an increasing function of μ (that you should solve for).

Solution: This question refers to the Cole Obstfeld 1991 article and the lecture on the benefits of financial integration. The social planner's problem is

$$\max_{x, y, x^*, y^*} \mu u(x, y) + (1 - \mu)u(x^*, y^*)$$

subject only to the world resource constraints

$$x + x^* = X$$

$$y + y^* = Y$$

The necessary first order conditions are

$$\mu u_1(x, y) = \gamma$$

$$\mu u_2(x, y) = \gamma^*$$

$$(1 - \mu)u_1(x^*, y^*) = \gamma$$

$$(1 - \mu)u_2(x^*, y^*) = \gamma^*$$

which imply

$$\frac{u_j(x, y)}{u_j(x^*, y^*)} = \frac{1 - \mu}{\mu}$$

In words, the social planner chooses allocations such that the ratios of marginal utilities across the two countries are constant at each state and date. We can solve for the allocation explicitly using the Cobb Douglas utility assumption. Evaluating the partial derivatives, we get

$$\frac{x^\theta y^{-\theta} (x^\theta y^{1-\theta})^{-\sigma}}{x^{*\theta} y^{*-\theta} (x^{*\theta} y^{*1-\theta})^{-\sigma}} = \frac{x^{\theta-1} y^{1-\theta} (x^\theta y^{1-\theta})^{-\sigma}}{x^{*\theta-1} y^{*1-\theta} (x^{*\theta} y^{*1-\theta})^{-\sigma}} = \frac{1 - \mu}{\mu}$$

and thus

$$\frac{x}{x^*} = \frac{y}{y^*} = \kappa$$

from the first equality, and where

$$\kappa = \left(\frac{1 - \mu}{\mu} \right)^{-1/\sigma}$$

from the second equality. Finally, using the global resource constraints,

$$\frac{x^*}{X} = \frac{y^*}{Y} = \frac{1}{1 + \kappa}$$

$$\frac{x}{X} = \frac{y}{Y} = \frac{\kappa}{1 + \kappa}$$

so that

$$x = \omega X$$

$$y = \omega Y$$

$$x^* = (1 - \omega)X$$

$$y^* = (1 - \omega)Y$$

with

$$\omega = \frac{1}{1 + \left(\frac{1-\mu}{\mu}\right)^{1/\sigma}}$$

If $\mu = 1/2$ we have perfect pooling. In any case, consumption is perfectly correlated across countries because idiosyncratic risk is insured across countries.

- (b) Set up the competitive equilibrium under financial autarky and solve for the allocations. Show that the allocations are given by

$$\begin{aligned} x &= \theta X \\ y &= (1 - \theta)X/p \\ x^* &= \theta p Y \\ y^* &= (1 - \theta)Y \end{aligned}$$

and the relative price of good x in terms of good y is given by $p = \frac{1-\theta}{\theta} \frac{X}{Y}$

Solution: The objective functions of each country remain the same. The home country now faces the budget constraint

$$x + py = X$$

while the foreign country faces

$$x^*/p + y^* = Y$$

where p is the relative price of good y in terms of good x . With Cobb Douglas preferences, expenditure shares satisfy

$$(1 - \theta)x = \theta py$$

and thus

$$\begin{aligned} x &= \theta X \\ y &= (1 - \theta)X/p \\ x^* &= \theta p Y \\ y^* &= (1 - \theta)Y \end{aligned}$$

Finally, market clearing requires

$$x + x^* = \theta X + \theta p Y = X$$

so

$$p = \frac{1 - \theta}{\theta} \frac{X}{Y} \tag{1}$$

- (c) Is the competitive equilibrium Pareto efficient? Why (not)? Explain the significance of this result and why it obtains in this model.

Solution: The competitive equilibrium is Pareto efficient if there exists a welfare weight μ such that the competitive equilibrium allocations solve the social planner's problem with those welfare weights. We see that the competitive equilibrium allocations are a solution to the social planner's problem if

$$\theta = \frac{1}{1 + \left(\frac{1-\mu}{\mu}\right)^{1/\sigma}}$$

Note that with log utility ($\sigma = 1$) this simplifies to $\mu = \theta$, for example. If $\theta = 0.5$, then $\mu = 0.5$ and the competitive equilibrium is efficient and entails perfect pooling despite the absence of asset markets. Significance: For a regime of financial autarky we have no theoretical reason to believe that the allocations should be efficient. Yet they are - and thus there are no gains from having financial asset markets (and complete markets as in the social planner case, in particular). This result occurs due to terms of trade effects. As shown in (1), the relative price of the foreign good optimally increases in response to positive wealth shocks at home. In other words, price effects offset wealth effects, inducing risk sharing. In this special case, the two effects offset each other perfectly.

- (d) How would you expect the substitutability between goods x and y and the intertemporal elasticity of substitution to affect estimates of the welfare losses from financial autarky? Explain your reasoning.

Solution: They are nil under very specific circumstances - perfect specialization in production and identical Cobb Douglas utility as in the first part of this question. Cole Obstfeld 1991 consider a generalization in terms of the utility function to estimate the gains and find that the higher the elasticity of substitution between the two goods and the more risk averse households are the larger the gains from financial integration are likely to be. The latter is intuitive - a stronger preference for smooth consumption paths implies larger losses when there are small divergences from it. The former is intuitive when considering the limit of perfect substitutability between goods, for example. Then we are effectively back in a 1 good world, and the assumption of financial autarky implies full autarky. As long as then the shocks between the two countries are not perfectly correlated, ie as long as there is some idiosyncratic risk to insure, we will have welfare losses from not being able to do so. Having said this, gains are generally small in their estimates. A correct answer might also elaborate on the role of nontradable goods or other generalizations discussed in Cole Obstfeld 1991 that break their knife-edge result of no gains from financial integration.

- (e) How and why would you expect the result from part c) to change if there was in addition a third tradable good z that both countries value and that both coun-

tries are endowed with? You do not need to use math to answer this question, as long as you are precise with words instead. For concreteness you can assume $u(x, y, z) = x^{\theta_x} y^{\theta_y} z^{\theta_z}$ with $\theta_x + \theta_y + \theta_z = 1$ if you want.

Solution: Shocks to the supply of the common tradable good generate wealth effects such that the terms of trade effects in general do not provide perfect international risk sharing anymore. Specifically, consider a positive shock to the home country supply of z . This constitutes a positive wealth effect for the home country, their lifetime income increased unexpectedly. Since world supply of good z is now relatively abundant, its relative price vis-a-vis the other two goods falls. This negatively affects the foreign country for who it is now more expensive to import and consume x . Note how this is different from the case of perfect specialization where a positive shock in the home country is transmitted abroad “positively”: A positive shock at home raises the price of the foreign good, so the foreign country exclusively benefits in terms of price effects.

The more mathematical answer - not required! - is the following. Define the price of x and y relative to z respectively as p_x and p_y . Then the modified country budget constraints are

$$\begin{aligned} p_x x + p_y y + z &= p_x X + Z \\ p_x x^* + p_y y^* + z^* &= p_y Y + Z^* \end{aligned}$$

Given the Cobb Douglas assumption we have

$$\frac{p_x x}{\theta_x} = \frac{p_y y}{\theta_y} = \frac{z}{\theta_z}$$

so that

$$\begin{aligned} p_x x &= \theta_x (p_x X + Z) \\ p_y y &= \theta_y (p_x X + Z) \\ z &= \theta_z (p_x X + Z) \end{aligned}$$

Market clearing for X and Y implies

$$\begin{aligned} \theta_x (p_x X + p_y Y + Z + Z^*) &= p_x X \\ \theta_y (p_x X + p_y Y + Z + Z^*) &= p_y Y \end{aligned} \quad (2)$$

so that, adding, we have

$$(\theta_x + \theta_y)(Z + Z^*) = (1 - \theta_x - \theta_y)(p_x X + p_y Y) \quad (3)$$

Noting that $\theta_x + \theta_y = 1 - \theta_z$, dividing (2) by X and using (3) to substitute for $p_x X + p_y Y$ we can solve for the relative price of X as

$$p_x = \frac{\theta_x}{\theta_x} \frac{Z + Z^*}{X}$$

and analogously

$$p_y = \frac{\theta_y}{\theta_z} \frac{Z + Z^*}{Y}$$

Plug this back into to find the solutions for the demand for good x

$$x = X \left(\theta_x + \theta_z \frac{Z}{Z + Z^*} \right)$$

We see that the demand for x varies with the supply of Z , not just $Z + Z^*$. In general, perfect risk sharing is not possible. The intuition can be seen from the expressions for the relative prices: Both increase in response to a home increase in the supply of Z and the positive “productivity” shock is transmitted negatively abroad by making both x and y relatively more expensive.